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An informational cascade occurs when it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information. We argue that localized conformity of behavior and the fragility of mass behaviors can be explained by informational cascades.

Let them alone: they be blind leaders of the blind. And if the blind lead the blind, both shall fall into the ditch.
[Matthew 15:14]

I. Introduction

One of the most striking regularities of human society is localized conformity. Americans act American, Germans act German, and Indi-

We thank Daniel Asquith, Christopher Barry, Gary S. Becker, Michael J. Brennan, Colin Camerer, Darrell Duffie, Robert Ellickson, Dan Friedman, Stephen J. Gould, Jack Hirshleifer, David Hull, Timur Kuran, Steve Lippman, John Mamer, Hubert Markl, Robert May, David Nachman, Jonathan Paul, Tom Philipson, Ivan P. L. Png, Thomas Schelling, Max Steuer, Sheridan Titman, Gordon Tullock, the editor, two anonymous referees, the members of the Rational Choice seminar at the University of Chicago, and seminar participants at University of California, Berkeley, Hebrew University, Jerusalem, the National Bureau of Economic Research, National Taiwan University, Northwestern University, the University of Pittsburgh, and the University of Utah for helpful comments and discussions. Parts of this paper subsume parts of an earlier paper, "Sequential Sales, Path Dependence, and Cascades" (September 1989 version) by Welch.

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ans act Indian. At one school teenagers take drugs, but at another they “just say no.” English and American youths enthusiastically enlisted to fight in World War I, but pacifist sentiments prevailed prior to World War II and in the 1960s.

Four primary mechanisms have been suggested for uniform social behavior: 1 (1) sanctions on deviants, 2 (2) positive payoff externalities, 3 (3) conformity preference, 4 and (4) communication. 5 The first three theories can explain why society may fix on undesirable choices, or at least why the social outcome may be history-dependent. Sanctions can enforce a malevolent dictatorship, payoff externalities can drive a better technology to extinction (e.g., the beta video system), and people with a direct preference for conforming may jump on the bandwagon for fairly arbitrary behavior (e.g., bell-bottom jeans). These effects tend to bring about a rigid conformity that cannot be broken by small shocks. Indeed, the longer the bandwagon continues, the more robust it becomes. The fourth theory implies convergence toward the correct outcome if communication is credible and costless. It does not explain why mass behavior is error-prone.

None of these theories explains why mass behavior is often fragile in the sense that small shocks can frequently lead to large shifts in behavior. 6 For example, cohabitation of unmarried couples was viewed as scandalous in the 1950s, was flaunted in the 1960s, and was hardly noticed in the 1980s. Colleges in which students demonstrated and protested in the 1960s became quiet in the 1980s. The recent rejection of communism began in Poland and later spread rapidly among other Eastern European countries. Religious movements, revivals, and reformations, started by a few zealots, sometimes sweep across populations with astonishing rapidity. Addiction to and social attitudes associated with alcohol, cigarettes, and illegal drugs have fluctuated widely.

This paper offers an explanation not only of why people conform but also of why convergence of behavior can be idiosyncratic and

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1 Boyd and Richerson (1985) examine several general models of cultural transmission that could be consistent with these mechanisms. Becker (1991) analyzes conformity in product demand in a model that is also consistent with several of these mechanisms.


3 See, e.g., Schelling (1960, 1978), Dybvig and Spatt (1983), Farrell and Saloner (1986), Katz and Shapiro (1986), and Arthur (1989). For example, conventions such as driving on the right- (or left-) hand side of the road are self-enforcing, once a few individuals follow the convention.

4 In Jones (1984), individuals inherently wish to conform with the behavior of others.

5 Conformity can be achieved if early individuals explain the benefits of alternatives to later ones (see, e.g., Rogers 1983).

6 In Kuran (1989), sanction-enforced behavior for specific sets of exogenous parameter values can be sensitive to small shifts.
fragile. In our model, individuals rapidly converge on one action on the basis of some but very little information. If even a little new information arrives, suggesting that a different course of action is optimal, or if people even suspect that underlying circumstances have changed (whether or not they really have), the social equilibrium may radically shift. Our model, which is based on what we call "informational cascades," explains not only conformity but also rapid and short-lived fluctuations such as fads, fashions, booms, and crashes. In the theories of conformity discussed earlier, small shocks lead to big shifts in mass behavior only if people happen to be very close to the borderline between alternatives. Informational cascades explain why society, on the basis of little information, will systematically tend to land close to the borderline, causing fragility.

An informational cascade occurs when it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information. Consider the submission of this paper to a journal. The referee will read the paper, assess its quality, and accept or reject it. Suppose that a referee at a second journal learns that the paper was previously rejected. Under the assumption that the referee cannot assess the paper's quality perfectly, knowledge of the prior rejection should tilt him toward rejection. Suppose now that the second journal also rejects and that when the paper is submitted to a third journal, the third referee learns that the paper was rejected at two previous journals. Clearly, this further raises the chance of rejection.

In a fairly general setting with sequential choices, we show that at some stage a decision maker will ignore his private information and act only on the information obtained from previous decisions. Once this stage is reached, his decision is uninformative to others. Therefore, the next individual draws the same inference from the history of past decisions; thus if his signal is drawn independently from the same distribution as previous individuals', this individual also ignores his own information and takes the same action as the previous individual. In the absence of external disturbances, so do all later individuals.

The paper submission example is special in that only one journal can accept the paper, ending the submission process. Thus the only possible cascade that can arise is one of rejection. In many situations cascades can be either positive—wherein all individuals adopt—or negative—wherein all individuals reject. Consider a teenager deciding whether or not to experiment with drugs. A strong motive for experimenting with drugs is the fact that friends are doing so. Conversely, seeing friends reject drugs could help persuade a youth to stay clean.
Although the outcome may or may not be socially desirable, a reasoning process that takes into account the decisions of others is entirely rational even if individuals place no value on conformity for its own sake. Imitation is, of course, an important social phenomenon, as has been documented by numerous studies in zoology, sociology, and social psychology. Our contribution is to model the dynamics of imitative decision processes as informational cascades.

We examine (1) how likely it is that a cascade occurs, (2) how likely it is that the wrong cascade occurs (can a good paper be unpublishable?), (3) how fashions change (why were college students of the 1980s pre-business "achievers," whereas those in the 1960s flirted with "alternative cultures"?), and (4) how effective are public information releases (e.g., a campaign to publicize the health effects of smoking).

There are several related papers in which private information causes individuals to imitate the actions of others. In Conlisk's (1980) evolutionary model, optimizers—who incur a decision cost—coexist with imitators—who avoid this cost but make inferior decisions because of observational lags. Welch (1992) examines the likelihood of cascades and optimal pricing in the market for initial public stock offerings. Banerjee (in press) independently models "herd behavior" as cascades. Conceptually, our paper differs from Welch's and Banerjee's in emphasizing the fragility of cascades with respect to different types of shocks; cascades can explain not only uniform behavior but also drastic change such as fads.

The remainder of the paper is structured as follows. Section II presents the basic model, shows that cascades can often be mistaken, and provides conditions under which a cascade will almost surely start. It then examines how a few early individuals can have a disproportionate effect and how small parameter shifts can transform an imitator into a fashion dictator. Section III examines the effect of prior disclosure of public information and shows that cascades are fragile when new public information can arrive. Section IV discusses several examples. Section V shows how the possibility of changes in the underlying value of alternative decisions can lead to "fads," that is, to drastic and seemingly whimsical swings in mass behavior without obvious external stimulus. Section VI concludes the paper.

7 Becker (1991) examines pricing decisions under demand externalities that might arise from informational sources.
8 In Scharfstein and Stein (1990) and Zwiebel (1990), conformity is an agency phenomenon. Scharfstein and Stein show that a manager may imitate the action of a preceding manager in order to improve his reputation for high ability. Zwiebel shows that relative performance evaluation may cause managers to adhere to inferior industry standards. Bhattacharya, Chatterjee, and Samuelson (1986) provide an interactive learning model of research and development.
II. The Basic Model

Information transmission among individuals can take many forms. For example, individuals may observe all other individuals' information, only the signals of predecessors, or only the actions of predecessors. Our analysis concentrates on the least informative case in which individuals observe only the actions of previous individuals. Since "actions speak louder than words," the information conveyed by actions may also be the most credible.

A. A Specific Model

For expository clarity, we begin with a specific model. Assume that there is a sequence of individuals, each deciding whether to adopt or reject some behavior. Each individual observes the decisions of all those ahead of him. The ordering of individuals is exogenous and is known to all.\(^9\) All individuals have the same cost of adopting, \(C\), which for now we set to \(\frac{1}{2}\). The gain to adopting, \(V\), is also the same for all individuals and is either zero or one, with equal prior probability \(\frac{1}{2}\).\(^{10}\) Individuals differ in their positions in the queue. Each individual privately observes a conditionally independent signal about value. Individual \(i\)'s signal \(X_i\) is either \(H\) or \(L\), and \(H\) is observed with probability \(p_i > \frac{1}{2}\) if the true value is one and with probability \(1 - p_i\) if the true value is zero. Table 1 describes this binary signal case.

We examine the special case of identically distributed signals (\(p_i = p\) for all \(i\)). The expected value of adoption is just \(E[V] = \gamma \cdot 1 + (1 - \gamma) \cdot 0 = \gamma\), where \(\gamma\) is the posterior probability that the true value is one. As a tie-breaking convention, an individual indifferent between adoption and rejection adopts or rejects with equal probability.

Thus the first individual adopts if his signal is \(H\) and rejects if it is \(L\). The second individual can infer the first individual's signal from his decision. If the first individual adopted, the second individual adopts if his signal is also \(H\). However, if his signal is \(L\), the second individual computes the expected value of adoption (given one \(H\) and one \(L\) signal) to be \(\frac{1}{2}\). Being indifferent, he adopts with probability \(\frac{1}{2}\). Similarly, if the first individual had rejected, the second individual rejects if his signal is also \(L\) and accepts with probability \(\frac{1}{2}\) if his signal is \(H\). The third individual is faced with one of three situations: (1) both predecessors have adopted (in which case even an \(L\) signal induces him to adopt and thus creates an upward cascade), (2) both have

\(^9\) In Sec. IIC, we briefly discuss the determination of the order of moves.

\(^{10}\) The model also applies to the choice between two arbitrary actions, where \(V\) and \(C\) are the differences in values and in costs.
TABLE 1

|                  | $\Pr(X_i = H|V)$ | $\Pr(X_i = L|V)$ |
|------------------|------------------|------------------|
| $V = 1$          | $p_i$            | $1 - p_i$        |
| $V = 0$          | $1 - p_i$        | $p_i$            |

rejected (in which case even an $H$ signal induces him to reject and thus creates a down cascade), or (3) one has adopted and the other rejected. In the last case, the third individual is in the same situation as the first individual: his expected value of adoption, based only on his predecessors’ actions, is $\frac{1}{2}$, and therefore his signal determines his choice. Should this come about, then a similar analysis shows that the fourth individual would be in the same situation as the second individual, the fifth as the third, and so forth.

With this decision rule, we can derive the unconditional ex ante probabilities of an up cascade, no cascade, or a down cascade after two individuals,

$$\frac{1 - p + p^2}{2}, \quad p - p^2, \quad \frac{1 - p + p^2}{2},$$

and

$$\frac{1 - (p - p^2)^{n/2}}{2}, \quad (p - p^2)^{n/2}, \quad \frac{1 - (p - p^2)^{n/2}}{2} \quad (1)$$

after an even number of individuals $n$.\(^{11}\) Equation (1) shows that the closer $p$ is to $\frac{1}{2}$, the later a cascade is likely to start. A reduction in $p$ toward $\frac{1}{2}$ is equivalent to adding noise to the signal; at $p = \frac{1}{2}$, the signal is uninformative. In other words, cascades tend to start sooner when individuals have more precise signals of the value of adoption.\(^{12}\) Moreover, according to (1), the probability of not being in a cascade falls exponentially with the number of individuals. Even for a very

\(^{11}\) After two individuals, no cascade occurs if there is one $H$ and one $L$. This value can be calculated assuming $V = 1$ (or 0). The occurrence of either $HL$ or $LH$ involves a coin flip, so the total probability is $\frac{1}{2}p(1 - p) + \frac{1}{2}p(1 - p) = p(1 - p)$. For the other two values, it suffices to note that since these probabilities are not conditional on $V$, $\Pr(\text{up}) = \Pr(\text{down}) = \frac{1}{2}[1 - \Pr(\text{no cascade})]$. For the expressions in eq. (1), note that the probability of an up cascade after four individuals is the probability of an up cascade after two individuals plus the probability of not being in a cascade after two individuals multiplied by the probability of an up cascade after another two individuals. In contrast, the probability of not being in a cascade after four individuals is simply the probability of not being in a cascade after two individuals multiplied by the probability of not being in a cascade after another two individuals.

\(^{12}\) Specifically, higher-precision $p$ raises the probability of histories that lead to the correct cascade.
noisy signal, as when \( p = \frac{1}{2} + \epsilon \), with \( \epsilon \) arbitrarily small, this probability after only 10 individuals is less than 0.1 percent!

We can also derive the probability of ending up in the correct cascade. The probabilities of an up cascade, no cascade, or a down cascade after two individuals, given that the true value is one, are

\[
\frac{p(p + 1)}{2}, \quad p(1 - p), \quad \frac{(p - 2)(p - 1)}{2}
\]

and after an even number of individuals \( n \) are

\[
\frac{p(p + 1)[1 - (p - p^2)^{n/2}]}{2(1 - p + p^2)}, \quad (p - p^2)^{n/2},
\]

\[
\frac{(p - 2)(p - 1)[1 - (p - p^2)^{n/2}]}{2(1 - p + p^2)}.
\]

The first expression is the probability of the correct cascade. It can be shown that this probability is increasing in \( p \) (see fig. 1) and \( n \). Even for very informative signals (where \( p \) is far from \( \frac{1}{2} \)), the probability of the wrong cascade is remarkably high.

The problem with cascades is that they prevent the aggregation of information of numerous individuals. Ideally, if the information of many previous individuals is aggregated, later individuals should converge to the correct action. However, once a cascade has started,
actions convey no information about private signals; thus an individual’s action does not improve later decisions.

Rogers and Shoemaker (1971) summarize research on the ability of outsiders (or “change agents”) to bring about the adoption of desirable innovations within communities. They offer the general proposition that “change agent success is positively related to his efforts in increasing his clients’ ability to evaluate innovations” (p. 247). This is consistent with the prediction of this binary example that as the precision of the signal, \( p \), increases, a correct cascade starts with higher probability and, on average, earlier.

It is instructive to compare the outcome in the previous-actions-observable (PAO) regime to that of the more informative previous-signal-observable (PSO) regime. In the binary signal case, PAO leads to a more uniform outcome. Following any given sequence of signal realizations, the two regimes lead to precisely identical outcomes, until a cascade begins in the PAO regime. However, in the PAO regime, after a cascade starts it is never reversed. In the PSO regime, even if an individual does not follow his private signal, it joins the common pool of knowledge. Hence, a long enough series of opposing signals will eventually cause people’s behavior to switch. Thus the PAO leads to greater uniformity. We shall argue in Section V that this uniformity is brittle: small shocks can easily shift the behavior of many individuals.

B. A General Model

We now show that under mild assumptions on the signals and values, cascades will always arise. Let there be a sequence of individuals \( i = 1, 2, \ldots, n, \ldots \), each deciding whether to adopt some behavior or to reject it. Each individual observes the decisions of all those ahead of him. The order of individuals is exogenous and is known to all. All individuals have the same cost of adopting, \( C \), and gain to adopting, \( V \). The gain \( V \) has a finite set of possible values, \( v_1 < v_2 < \ldots < v_3 \), and the decision is not trivial \( (v_1 < C < v_3) \). The prior probability that \( V = v_i \) is denoted \( \mu_i \).

We use the concept of perfect Bayesian equilibrium. Since an individual’s payoffs do not depend on what later individuals do, there is no incentive to make an out-of-equilibrium move to try to influence a later player. Thus, without loss of generality, we assume that if any player is observed to deviate from the equilibrium, either by rejecting when he should have adopted regardless of his signal realization or by adopting when he should have rejected regardless, then subsequent individuals have the same beliefs as though he had chosen his correct (equilibrium) action.
Individuals differ not only by their positions in the queue but by the signals they privately observe. Each individual \( i \) observes one of a conditionally independent and identically distributed sequence of signals \( X_i \) with possible values \( x_1 < x_2 < x_3 < \ldots < x_R \). Let \( P_{q,l} \) be the probability that an individual observes signal value \( x_q \) given a true value of adoption of \( v_l \). We assume that \( P_{q,l} > 0 \) for all \( q \) and \( l \). Let \( P_{q,l} \) be the cumulative distribution of \( X_i \) given \( V = v_l \), that is,

\[
P_{q,l} = \Pr(X_i \leq x_q | V = v_l) = \sum_{j=1}^{q} P_{j,l}.
\]

Let \( J_i \) be the set of signal realizations that lead individual \( i \) to adopt. His decision communicates to others that he observed either a signal in the set \( J_i \) or its complement. If \( J_i = \{x_1, x_2, \ldots, x_R\} \) or if \( J_i \) is empty, then individual \( i \)'s action conveys no information about his realization.

**Definition.** An informational cascade occurs if an individual's action does not depend on his private information signal.

If an individual \( i \) is in a cascade, then his action conveys no information and individual \( i + 1 \) draws the same inference from all previous actions. Since the signal \( X_{i+1} \) is drawn from the same distribution as \( X_i \), individual \( i + 1 \) is also in a cascade. Thus, by induction, all individuals after \( i \) are in a cascade. Consequently, a cascade once started will last forever, even if it is wrong. We shall see later that this fallibility causes cascades to be fragile. For instance, if individuals' signals have different distributions, as in Section IIIC, if public information is revealed at a later date, as in Section IIIA, or if underlying values can change, as in Section V, then cascades can easily be broken.

Let \( a_i \) be individual \( i \)'s action (adopt or reject) and let \( A_i = (a_1, a_2, \ldots, a_i) \) represent the history of actions taken by individuals \( 1, 2, \ldots, i \). Given history \( A_{i-1} \), let \( J_i(A_{i-1}, a_i) \) be the set of signal realizations that lead individual \( i \) to choose action \( a_i \). Then individual \( n + 1 \)'s conditional expectation of \( V \) given his own signal realization \( x_q \) and the history \( A_n \) is

\[
V_{n+1}(x_q; A_n) = E[V | X_{n+1} = x_q, X_i \in J_i(A_{i-1}, a_i), \text{ for all } i \leq n].
\]

Individual \( n + 1 \) adopts if\(^{15}\) \( V_{n+1}(x_q; A_n) \geq C \). Therefore, the inference drawn from \( n + 1 \)'s action \( a_{n+1} \) is that \( X_{n+1} \in J_{n+1}(A_n, a_{n+1}) \), where

\[
J_{n+1}(A_n, \text{adopt}) = \{x_q \text{ such that } V_{n+1}(x_q; A_n) \geq C\},
\]

\[
J_{n+1}(A_n, \text{reject}) = \{x_q \text{ such that } V_{n+1}(x_q; A_n) < C\}.
\]

\(^{15}\) Since we are no longer restricting ourselves to a symmetric example, we use a tie-breaking assumption (indifferent individuals adopt) slightly different from the one in the previous section (indifferent individuals randomize). This reduces notation but not generality.
We impose two regularity conditions. The first is that the conditional distributions \( \Pr(X_i | V = v_i) \) are ordered by the monotone likelihood ratio property.\textsuperscript{14} This ensures that if an individual observes a higher signal realization, he infers that the value of adoption is higher (in a first-order stochastic dominance sense).

**Assumption 1.** Monotone likelihood ratio ordering.—For all \( l < S \),

\[
\frac{p_{q,l}}{p_{q+1,l}} \geq \frac{p_{q,l+1}}{p_{q+1,l+1}} \quad \text{for all } q < R,
\]

with strict inequality for at least one \( q \).

Assumption 1 ensures that the conditional expectation of each individual increases in his signal realization. Thus if individual \( i \) is not in a cascade and he adopts, later individuals conclude that \( X_i \geq x_q \) for some \( q \). If \( i \) does not adopt, then the conclusion is that \( X_i < x_q \).

The second assumption ensures that if individuals learn enough about value by observing predecessors, then they are not indifferent between adopting and rejecting.\textsuperscript{15}

**Assumption 2.** No long-run ties.—\( v_l \neq C \) for all \( l \).

A major result of this section is that a cascade eventually begins. Suppose that an individual late in the sequence is still making a decision based on his own information. Then the decisions of earlier individuals convey some information about their signals. If this individual is far enough down the line, then, by the strong law of large numbers, with probability close to one he can infer the true value of adoption with almost perfect certainty. But then his own signal contributes arbitrarily little to his information set, and he acts according to the information conveyed by the actions of previous individuals. Therefore, he ignores his private information and starts a cascade.

**Proposition 1.** If assumptions 1 and 2 hold, then as the number of individuals increases, the probability that a cascade eventually starts approaches one.

While we have shown that cascades must eventually occur, perhaps the more interesting point is that they will often be wrong.\textsuperscript{16} In an

\textsuperscript{14} The monotone likelihood ratio property is a standard assumption in models in which inferences must be drawn from a noisy signal. Milgrom (1981) provides a presentation and applications.

\textsuperscript{15} It is a mild assumption. If \( v_1, v_2, \ldots, v_S \) and \( C \) are drawn randomly from any nonatomic probability measure, assumption 2 is satisfied with probability one. This assumption prevents asymptotic indifference but is not a tie-breaking convention.

\textsuperscript{16} The proof of proposition 1 is based on the idea that many observations of informative actions would lead with high probability to nearly perfect knowledge of value. Thus with high probability a cascade must start at or before such a point, but this cascade will often be an incorrect one that started much earlier. As fig. 1 in Sec. IIA shows, the probability of an incorrect cascade in the specific model can be close to .5 if the signal is noisy enough.
example with binary signals and a uniform prior on the true value, Welch (1992) has shown that cascades will start and can often be wrong. Banerjee (in press) reaches the same conclusion assuming a continuous uniform prior distribution on the correct action; however, incorrect cascades in his setting derive from a degenerate payoff function.  

\[17\]

C. Fashion Leaders

We now consider a scenario in which individuals have different signal precisions (accuracy). In particular, consider the binary signal case of table 1, where higher precision of individual i’s signal refers to a higher value of \( p_i \). We assume that \( \Pr(V = 1) = \Pr(V = 0) = \frac{1}{2} \).

**Result 1.** Suppose that the binary signal case obtains. (1) If \( C = \frac{1}{2} \) and if the individual with the highest precision decides first, then the first individual’s decision is followed by all later individuals. (2) Assume that all individuals \( n > 1 \) observe signals of identical precision. Then all individuals \( n > 2 \) are better off if the first individual’s precision is slightly lower rather than slightly higher than theirs.

**Proof.** (1) The second individual infers the first individual’s signal and so ignores his own information, starting a cascade. (2) If the first individual’s precision is slightly higher, the second individual defers to the first individual; if it is slightly lower, the second individual makes his own decision. Thus the latter case leads to more information for later individuals. Q.E.D.

Result 1 illustrates that small differences in precision can be very important and can lead to cascades that are even less informative (and, so, potentially even more fragile) than when individuals have identically distributed signals. While order is exogenous in the model, it is plausible that the highest-precision individual decides first. Consider a more general setting in which all individuals have the choice to decide or to delay, but there is a cost of delaying decision. All individuals have an incentive to wait in the hope of free-riding on the first to decide. However, other things equal, the cost of deciding early is lowest for the individual with the highest precision.

The fashion leader model applies to situations in which a veteran performs a task with novices. If an experienced individual acts first, others frequently imitate. The prediction that a low-precision individual imitates a higher-precision predecessor is consistent with the evidence of numerous psychological experiments demonstrating that a subject’s previous failure in a task raises the probability that in further

\[17\] As Lee (1991) shows, with a continuum of actions, behavior generically converges to the correct action.
trials he will imitate a model performing the task (see Thelen, Dollinger, and Kirkland 1979, p. 146). Deutsch and Gerard (1955) also give experimental evidence that the more uncertain an individual is about the correctness of his judgment, the more susceptible he is to informational influences on his decisions. Being better informed, teachers and parents are natural opinion leaders (see Ainlay, Becker, and Coleman 1986). Rogers (1983) summarizes studies that show that community leaders have superior information. Similarly, Stamps (1988, p. 340) summarizes evidence that among territorial animals, "individuals acquiring their first territory in an unfamiliar habitat are more apt to prefer territories next to previous settlers than would territory owners or floaters that had lived in the habitat in the past."

Result 1 illustrates that a very slight perturbation in the informational setting may make a very large difference (between an immediate or later cascade). Thus to understand the "cause" of a social change, it is crucial to pay careful attention to the early leaders. Indeed, when mass behavior arises idiosyncratically from chance early events, it can be futile to seek grand causal forces.

Result 1 also suggests that an individual who wishes to bring about a social change, for example, introduce a desirable innovation such as an improved sanitary method in a peasant community, must focus his efforts on persuading early community leaders. Assume, for example, that individuals are ordered by precisions. Suppose that the "change agent" can persuade by causing one individual's signal realization to be (correctly) high, that is, perfect information precision. Suppose, however, that others are not aware of the improved precision of the persuaded individual. Then the change agent should focus his efforts on the first and best-informed individual. Studies by Bliss (1952, p. 30) and Alers-Montalvo (1957, p. 6) find that individuals attempting to bring about social change are more successful when they work through community leaders (who tend to be better informed).19

Although we have suggested that higher-precision individuals tend to decide earlier, it is worth considering what occurs if a higher-

18 Rogers and van Es (1964) provide evidence that community leaders in Colombian peasant communities have more formal education, higher literacy, larger farms, higher social status, more exposure to mass media, and more political knowledge than followers (see also Rogers and Shoemaker 1971). Some social psychologists have recognized that imitation may be based on a belief that high-prestige individuals are good decision makers. Bandura (1977, p. 89) states that "in situations in which people are uncertain about the wisdom of modeled courses of action, they must rely on such cues as general appearances, speech, style, age, symbols of socioeconomic success, and signs of expertise as indicators of past successes."

19 Our prediction that the first individual is entirely decisive is often unrealistic. A milder result, that the first individual is disproportionately influential, could be derived under a more general information structure.
precision individual is later in the sequence. Such an individual can shatter a cascade because he is more inclined to use his own information than those that precede him. This possibility of cascade reversal tends to improve decisions because more information can be aggregated than if only a single cascade occurred. Thus from a social point of view, it may be desirable to order decision makers inversely with their precisions. It is not easy, however, to think of how such a regime would arise spontaneously.

III. Are Cascades Fragile?

We have argued that the actions of early individuals can influence the behavior of others so that later individuals ignore their own information and merely follow suit. The uniformity that cascades cause can be similar to and coexist with that brought about by the other forces discussed in the Introduction (sanctions, payoff externalities, and conformity preference). However, while the uniformity stemming from these other factors becomes more robust as the number of adopters increases, the “depth” of an informational cascade need not rise with the number of adopters; once a cascade has started, further adoptions are uninformative. Thus conformity is brittle. The arrival of a little information or the mere possibility of a value change (even if the change does not actually occur) can shatter an informational cascade.

A. The Public Release of Information

Cascades can be sensitive to public information releases. For example, behavior may reverse when government and research institutions release new information on the hazards of smoking and the effects of medical procedures (e.g., tonsillectomy), drugs (e.g., aspirin), and diet (e.g., oat bran). We address three questions in this subsection: (1) Does the single release of information make all subsequent individuals better off? For example, are all potential aspirin users better off if all are provided with more information about its effects on heart disease? (2) Can a cascade be reversed, and how difficult is this? For example, should the government release information to dissuade potential smokers from imitating the millions of addicted individuals? (3) Does the multiple release of information eventually make individuals better off? For example, if medical science gradually generates information about the adverse consequences of tonsillectomy without special medical indications, will all doctors eventually reject this practice?
1. The Effect of an Initial Public Disclosure

Result 2 addresses the first question.

RESULT 2. The release of public information before the first individual's decision can make some individuals worse off (in an ex ante sense).

Proof. See the Appendix.

The public information release has two effects on an individual: (1) it directly provides more information, and (2) it changes the decisions of predecessors and, thus, the information conveyed by their decisions. Result 2 is based on an example in which the public information is very noisy, but it reduces the information conveyed to the second individual by the first individual's decision so much that it outweighs the direct positive effect.

Thus it is by no means clear whether public health authorities should act quickly to disseminate noisy information. Sketchy disclosures of advantages of oat bran and fish oil, by triggering fads, may do more harm than good.\(^{20}\) On the other hand, a highly informative disclosure, such as the release of compelling evidence on the health effects of smoking, is likely to benefit everyone.

2. The Depth of a Cascade

The ambiguity of the effect of a public disclosure ceases when a cascade starts.

PROPOSITION 2. If all individuals' signals are drawn from the same distribution, then after a cascade has begun, all individuals welcome public information.

Proof. After a cascade has begun, if there is no public disclosure, individuals' decisions convey no further information. Thus public disclosure conveys information directly without reducing the information conveyed by individuals' decisions. Q.E.D.

Result 3 suggests that cascades are delicate with respect to new information.

RESULT 3. The release of a small amount of public information can shatter a long-lasting cascade, where a "small amount" refers to a signal less informative than the private signal of a single individual.

Proof. Consider the binary signal example of the previous section. An up (down) cascade ensues as soon as an individual observes two

\(^{20}\) Early medical reports indicated that oat bran lowers cholesterol levels, which suddenly increased the popularity of oat bran products; a new study contradicted this result, killing the fad. However, a subsequent study suggested that oat bran is moderately effective after all (see Consumer Reports Health Letter, April 1991, p. 31).
more adopt (reject) than reject (adopt) decisions. All subsequent individuals understand that either both individuals received $H$ ($L$) signals or only the first individual observed an $H$ ($L$) signal and the second flipped a coin. Consider a public signal that is slightly less informative than a private signal. Even if the cascade was due to two $H$'s, one public $L$ signal suffices to induce an individual to consider his information, since with one more $L$ signal he is indifferent between adopting and rejecting. Since there is a positive probability that the cascade was caused by only one $H$ ($L$) signal, the individual with public information thus acts according to his own information and adopts on $H$ and rejects on $L$. Q.E.D.

Intuitively, cascades aggregate the information of only a few early individuals' actions. The public information thus needs only to offset the information conveyed by the action of the last individual before the start of the cascade, even if millions subsequently imitated. Thus the fact that a vast segment of society already smokes need not discourage investigation of the effects of smoking.

A possible illustration of fragility is the adoption of hybrid corn by Iowa farmers from 1928 to 1941. Ryan and Gross (1943) interviewed Iowa farmers and found that the average time between first learning of hybrid corn and adopting was 9 years. Thus, for most of the period, respondents were aware of but did not adopt hybrid corn (down cascade). It seems likely that the later widespread adoption of hybrid corn (up cascade) was due to the arrival of further information about its effectiveness.21

3. The Effect of Multiple Public Information Disclosures

Since a cascade can be shattered by even a minor public information release, a relevant question is whether, asymptotically (as the number of disclosures becomes large), society is certain to settle into the correct cascade.22

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21 Ryan and Gross also found that neighbors' adoption was the most frequent reason for adoption, consistent with a cascade scenario. Similarly, Deutschmann and Fals Borda's (1962) study of Colombian peasant communities also suggests that cascades may be important for adoption of innovations. They found that villagers seldom tried farming innovations on a partial basis and that about 80 percent of respondents adopted fully after observing the use of the innovation on a neighbor's farm. It may be plausibly argued that in this setting individuals observe only their neighbors and not the entire history. However, in the specific model of Sec. IIA, even if individuals have imperfect recall in that they observe only the most recent predecessors, cascades can occur and will be fragile.

22 We follow the tie-breaking assumption of Sec. IIB that when indifferent the individual adopts.
PROPOSITION 3. If there is a probability, bounded from zero, of further public information release before each individual chooses (and the public information is conditionally independent and identically distributed and assumptions 1 and 2 are satisfied), then individuals eventually settle into the correct cascade.

The proof is a standard application of the law of large numbers and is omitted. It relies on the fact that as the number of public information releases increases, the correct choice becomes clearer. The strong law of large numbers ensures that as long as public information is conditionally independent and identically distributed, the posterior concentrates on the true value and each individual almost surely decides correctly. Thus each individual acts like the previous individual, and the correct cascade results.

Since proposition 3 relies on asymptotic arguments, it provides only moderate grounds for optimism. Further intuition can be gained from a numerical example. Consider again the binary-signal/value case discussed in Section IIA. However, we now introduce a small probability that an information signal, drawn independently from the same distribution as each individual's signal, is publicly released. Columns 1–2, 4–5, and 7–8 in table 2 list the probabilities that an UP cascade and a DOWN cascade will be in process when the 1,000th individual is reached as a function of \( p \), the probability that the signal is \( H \) given that the actual value of \( V \) is one. The probability of settling into the correct UP cascade increases dramatically even when only very few public releases of information occur on average. For example, if \( p = .75 \), the probability of ending up in the correct cascade increases from .81 when there is no public information release to .86 (.98) when on average one (10) release(s) of public information occurs per 1,000 individuals.

As a possible case in which an incorrect cascade started and then reverted to the correct cascade, Apodaca (1952) documented the introduction of one variety of hybrid seed corn for 84 growers in a New Mexico village from 1945 to 1949 in which a trend reversed before settling on an outcome. Since the hybrid seed yielded three times as much as the old seed, the percentage of adopters rose from 0 percent in 1945 to 60 percent in 1947. However, 2 years later it fell back to 3 percent when the villagers decided that the hybrid corn tasted worse.

Columns 3, 6, and 9 record the expectation of the difference between the number of inferred \( H \) and \( L \) signals after 1,000 individuals. With public information, the average cascade is quite deep and correctly positive. If \( p = .65 \) (a rather noisy signal), with 10 per 1,000 public information releases, the expected difference is 4.6.
| $p$ | **NO PUBLIC INFORMATION** | | **PUBLIC INFORMATION (1)** | | **PUBLIC INFORMATION (10)** |
|-----|--------------------------|-----|--------------------------|-----|
|     | Cascade                  | Expectation of Difference | Cascade | Expectation of Difference | Cascade | Expectation of Difference |
|     | Up (1)       | Down (2) | Difference (3) | Up (4) | Down (5) | Difference (6) | Up (7) | Down (8) | Difference (9) |
| .55 | .5645        | .4355   | .262        | .5857  | .4132  | .451        | .6767  | .3200  | 1.691        |
| .65 | .6950        | .3050   | .783        | .7396  | .2601  | 1.236       | .8986  | .1005  | 4.577        |
| .75 | .8100        | .1900   | 1.242       | .8633  | .1366  | 1.950       | .9826  | .0172  | 6.874        |
| .85 | .9065        | .0935   | 1.623       | .9480  | .0516  | 2.494       | .9989  | .0011  | 8.940        |
| .95 | .9724        | .0276   | 1.890       | .9884  | .0116  | 2.858       | 1.0000 | .0000  | 10.982       |
B. Discussion

The social cost of cascades is that the benefit of diverse information sources is lost. Thus a cascade regime may be inferior to a regime in which the actions of the first $n$ individuals are observed only after stage $n + 1$. However, the fragility of cascades allows some of the benefit of information diversity to be recaptured. Incorrect decisions, once taken, can be rapidly reversed. For instance, a high-precision individual late in the sequence can break a cascade, which leads to better decisions. Public information disclosures can break incorrect cascades and eventually bring about the correct decision.

IV. Examples

We have argued that in situations in which individuals with private information make sequential decisions, cascades may be pervasive. This section discusses some illustrative examples. Even though in many cases other factors (sanctions, payoff externalities, or conformity preference) may be present as well, the sequential process of decisions under uncertainty can lead to cascades.

We used several criteria for selecting examples. The first group of criteria pertains to model assumptions: (i) actions are sequential, (ii) decision makers combine their private information signals with those of previous individuals, (iii) decision makers act on the basis of observation of actions rather than verbal communication, and (iv) sanctions and externalities that might enforce uniformity are absent. Assumption i does not require a perfect linear ordering of individuals. As long as there is enough sequentiality in the model, results of a similar nature would apply.\(^{23}\) We view assumption iii as less important since "actions speak louder than words." With regard to assumption iv, externalities can oppose uniformity, strengthening the inference that some other effect (such as informational cascades) is the cause of uniformity. (However, even when externalities support uniformity, cascades may still play a role in determining which action will prevail.)

The second group of criteria pertains to model implications: (i) the phenomenon is local or idiosyncratic (in the sense that actions seem to have low correlation with the underlying desirability of the alterna-

\(^{23}\) Similar results also apply when individuals observe the actions of only a few immediate predecessors (see n. 21) and when individuals observe only a summary statistic for previous actions (e.g., aggregate sales figures). For example, in the specific model of Sec. IIA, an individual need only observe a public summary statistic, the difference between the number of adoptions and rejections, which substitutes perfectly for knowledge of the entire history.
tives), (ii) the phenomenon displays fragility, and (iii) some individuals ignore their own private information.

A. Politics

In a study of U.S. presidential nomination campaigns, Bartels (1988, p. 110) discusses "cue-taking," in which an individual's beliefs about a candidate are influenced by the decisions of others: "the operative logic is, roughly, that '25,000 solid New Hampshirites (probably) can't be too far wrong.' " Several studies of political momentum use numerical survey measures (called "thermometer ratings") of how much respondents like the candidate. These studies demonstrate that, when one controls for other factors, more favorable poll results cause respondents to evaluate a candidate more positively (Bartels 1988). This is consistent with informational cascades. Strategic voting, in which an individual's willingness to vote for a candidate depends on his expectation of the candidate's prospects, explains why early successes of a candidate may lead people to vote for him, but does not explain why his thermometer ratings should increase.

As a possible example, in the 1976 presidential campaign, the little-known candidate Jimmy Carter achieved an important early success by concentrating his efforts toward securing the Democratic nomination in the Iowa caucus (which preceded the first primary in New Hampshire). "Super Tuesday," in which many southern states coordinated their primaries on the same date, was an attempt to avoid the consequences of sequential voting.²⁴

B. Zoology

There is evidence of imitative behavior transmission among animals, especially in territory choice, mating, and foraging. According to Galef (1976, p. 78), "intraspecific interaction resulting in the transmission of acquired patterns of behavior from one individual to another within a population is a relatively common and important mode of adaptation in both primate and nonprimate vertebrate organisms." An advantage of zoological examples is that animals are less able to discuss the merits of alternative actions and are not influenced by "just say no" mass-media campaigns.²⁵

²⁴ In McKelvey and OrdeshooK's (1985) model, opinion polls convey information that causes bandwagons.
²⁵ Examples of behavioral spread through imitation of one individual by another include potato washing by Japanese macaques (Kawai 1965) and milk bottle opening by various species of tit in Britain.
Stamps (1988) discusses that, even after one controls for site quality, animals often arrange their territories in clusters. Clustering occurs despite possible negative externalities of being close to a competitor for mates and food. Persson's (1971) study of the whitethroat (Sylvia communis) found that clustering results because newcomers prefer to locate near earlier settlers. Consistent with our theory, Kiester and Slatkin (1974) argue that clustering can be explained by males' use of the presence of other males as an indicator of high resource quality in nearby territories. The cascades model suggests that the somewhat arbitrary choices of a few early settlers determine the locations of clusters. Several studies indicate that territorial clumping is idiosyncratic and is not mainly due to convergence on high-quality territories (see Stamps [1988] and the references therein).

In "How to Find the Top Male," Pomiankowski (1990) discusses recent evidence that females copy the mating choice of other females in lek species. For example, "in both fallow deer and sage grouse, the rate at which females enter male territories correlates with the number of females already present" (p. 66). In an experiment, when a stuffed female grouse was put on the territory of an unattractive male, the number of females entering the territory increased.

Gibson, Bradbury, and Vehrencamp (1991) establish the prevalence of copying among sage grouse after controlling for several other factors that could affect male reproductive success. Following Wade and Pruett-Jones (1990), they point out that copying explains why mate choice is highly unanimous even when mate choice correlates poorly with observable characteristics of males or their sites. This is consistent with the arbitrariness of group behavior implied by cascades. Copying here is particularly significant in view of some negative externalities: females must often wait long periods of time, and sperm may be depleted.

C. Medical Practice and Scientific Theory

Taylor (1979) and Robin (1984) discuss numerous surgical fads and epidemics of treatment-caused illnesses ("iatroepidemics"). Some operations that have come and gone in popularity are tonsillectomy, elective hysterectomy, internal mammary ligation, and ileal bypass. They argue that the initial adoption of these practices was frequently based on weak positive information. Robin also points out that most doctors are not well informed about the cutting edge of research; this

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26 In lek species such as fallow deer and sage grouse, females visit aggregations of males (the "lek") to select a mate. Some males mate with many females, whereas others do not mate.

27 Another example is physicians' use of leeches until the nineteenth century.
suggests that when in doubt, they may imitate. Burnum (1987) refers to “bandwagon diseases” popularized by physicians who, “like lemmings, episodically and with a blind infectious enthusiasm [push] certain diseases and treatments primarily because everyone else is doing the same” (p. 1222). In our model, one adoption may be enough to start a cascade. Therefore, when one obtains a second opinion, withholding the first opinion from the second doctor may be advisable.

We now compare the adoption of one surgical procedure, tonsillectomy, with the predictions of the cascades model. As Robin (1984, p. 75) points out, “For many decades, tonsillectomy was performed in millions of children on a more or less routine basis. In most cases, the operation was unnecessary.” He also states that tonsils are needed for defense against infections and that some children were injured and died during the procedure. The adoption of tonsillectomy was not associated with any definitive public information, such as controlled studies, supporting it (see Taylor 1979). A critical English panel, the Schools' Epidemic Committee of the Medical Research Council, claimed that tonsillectomy was being “performed as a routine prophylactic ritual for no particular reason and with no particular result” (quoted in Taylor [1979, p. 159]). The rate of tonsillectomy has declined in recent years.

There has also been significant idiosyncratic geographical variation in the frequency of tonsillectomy (Taylor 1979). In England in 1938, the frequency in some regions was one-half of the nation’s average, and in others it was three times as high. The only factor that could be implicated was variation in “medical opinion.” In the late sixties to seventies, tonsillectomy was six times more frequent in New England than in Sweden, with England and Australia in between.

Adoption of a scientific theory can also cascade. Very few people have carefully examined the evidence that the earth is round (e.g., Foucault's pendulum or anomalies on maps). But since many others have adopted the view, others accept it. Even among physicists, few can examine carefully the evidence on all major theories. Inevitably, individuals must accept the overall decisions of others rather than their arguments and evidence. Thus we expect the adoption of a theory to depend on the reputation of its early exponents.

**D. Finance**

In the market for corporate control, the arrival of a first takeover bid frequently attracts competing bids, despite the fact that the presence of the first bidder drives up the price. This suggests that the positive information conveyed by the first bidder’s putting the target “in play”
outweighs the negative payoff externality. On a larger scale, financial and takeover markets have waves, such as the conglomerate merger wave of the 1960s and its reversal through restructuring and hostile takeovers in the 1980s, that are hard to explain entirely by fundamental factors.

Another possible application is the decision of investors to subscribe to an initial public offering. Welch (1992) uses a cascade model to show that if sufficiently many (few) individuals sign up early to receive shares, all (no) subsequent individuals follow their lead. He further considers the optimal pricing of shares to induce cascades of subscription.

If one creditor refuses to renegotiate debt with a distressed firm, others may also. Similarly, the start of a bank run can be viewed as a cascade in which small depositors fear for the solvency of a bank and act by observing the withdrawal behavior of other depositors (see Diamond and Dybvig 1983).28

It has frequently been argued that stock market price movements are caused by waves of investor sentiment. Although cascades may apply to bubbles or crashes, this is not captured directly by our model, since the cost of adopting (e.g., buying a stock) increases as a bubble forms (see Friedman and Aohi 1991; Camerer and Weigelt, in press).

E. Peer Influence and Stigma

"Peer pressure" is often invoked as an explanation for conformity; the term connotes coercion. Our theory offers an alternative explanation for the influence of peers: that individuals, especially those with little information or experience, obtain information from the decisions of others. For example, in the important Asch (1952) experiments on the comparison of line lengths by members of groups, conformity can be interpreted as information-based rather than coercive (see Deutsch and Gerard 1955).

Stigma is the negative typecasting of persons that are outside the norm of the social unit.29 Evidence described in Ainlay et al. (1986) suggests that stigma is local (i.e., group-specific) and is learned by observing the behavior of others such as parents (see also Sigelman and Singleton 1986, p. 188). An individual may be stigmatized when negative information conveyed by earlier rejections starts a down cascade. For example, gaps in a job seeker's resume may reveal that

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28 Clearly, there are strong payoff externalities in both these examples. However, information about either fundamentals or the likelihood of a run may be conveyed by the early decision makers even if the first few decisions create little externality.

29 Akerlof (1976) provides a model of stigma based on ostracism.
potential employers chose not to hire him. Similarly, individuals who divorced or severed business ties may carry a stigma.

Conversely, a job applicant who receives early job offers may become a "star." In the rookie market for professors, later schools may give job interviews and offers based on the known interest of earlier schools. Similarly, to be granted tenure at a university, it is helpful to receive tenured offers elsewhere.\(^{30}\) Our theory suggests that an individual's early-career status as a star can be precarious.\(^{31}\)

V. Fads

Fads and conventions often change without apparent reason. In our basic model, cascades can cause individuals to converge on the wrong decision. We now propose that seemingly whimsical shifts in behavior occur because an initial cascade may aggregate very little information. If there is a small probability that the underlying value changes at a particular stage, then cascades can switch, not just because the right action has changed, but because people are not sure whether it has changed. The possibility that value changes can cause random signal outcomes to deceive, so that sometimes behavior changes even when value does not. Thus behavior may change frequently even if value seldom changes.

Although we analyze an example, we believe that similar results would apply in more general settings. Let the initial value of adoption be \( V = 0 \) or 1, both equally likely. This value of adoption remains unchanged until \( i = 100 \). At \( i = 101 \), with probability \(.1\) the true value may be redrawn (again with both zero and one being equally likely). Let \( W \) denote the true value after \( i = 100 \). Thus \( W = V \) with probability \(.95\), and \( W = 1 - V \) with probability \(.05\). The cost of adoption is \( C = .5 \).

Each individual \( i \) observes a signal \( X_i = H \) or \( L \) that is independently distributed conditional on the current true value as in table 1, with probability \( p_i = p \). We shall show that a fad can change even though the true value remains unchanged and that if \( p = .9 \), the probability of a change in behavior is greater than the probability that the value actually changes.

We describe three alternative regimes: full information, previous signals observable, and previous actions observable.

\(^{30}\) Rosen (1981) provides an alternative explanation for superstars based on scale economies that magnify the effect of skill differences.

\(^{31}\) Blanchard and Summers (1988) emphasize that the instability of unemployment rates implies a need for theories of fragile equilibrium in labor markets, and they discuss models based on downward-sloping labor supply curves and upward-sloping demand curves.
A. Full-Information Regime and Previous-Signals-Observable Regime

Under full information, all individuals can observe $V$ and always choose the correct action. Thus individuals 1–100 adopt if and only if $V = 1$, and individuals 101 and beyond adopt if and only if $W = 1$. At $i = 101$, behavior changes with probability .05.

Under the previous-signals-observable regime, the pattern of behavior is similar. Since the aggregate information of individuals rapidly becomes very precise, individuals 10 (say) through 100 choose the correct action with very high probability. Just before individual 101, a new value may be drawn; again by individual 110 the correct decision is almost surely taken. Hence, the probability of a change in behavior is very close to .05.

B. Previous Actions Observable: The Cascade Regime

We now examine the probability of a change in behavior at $i = 101$ (i.e., the probability of going from an up cascade before $i = 101$ to a down cascade soon after $i = 101$, or vice versa) when only previous actions are observable.\footnote{As in Sec. IIA, we adopt the tie-breaking convention that if an individual is indifferent between adopting and rejecting, he adopts with probability $\frac{1}{2}$.} Equation (1) implies that with probability close to one, a cascade starts by period 100 and that an up and a down cascade are equally likely. We summarize this example as follows.

**Result 4.** In the numerical example on fads the probability of a cascade reversal is greater than .0935, compared to a probability of only .05 that the correct choice changes. Therefore, the probability of convention changes can be substantially higher when only previous actions are observed than under full information.

**Proof.** See the Appendix.

The probability of behavior shifts can be much higher (87 percent higher in this example) than under the full-information and the previous-signals-observable regimes. Intuitively, and in contrast with a full-information regime, individuals are not very confident that the original cascade was correct.

This is more an example of fads than of fashions, in that we still assume a true underlying value that is independent of the actions of participants. In clothing fashion, for example, whether “pink is in” or whether a short skirt is acceptable this season depends on who else decides to adopt the fashion.\footnote{Karni and Schmeidler (1990) and Matsuyama (1991) provide theories of fashion in which some individuals have a taste for being different from others. In Conlisk (1976), individuals have a preference for change per se. In Bulow and Klemperer (1991), information about a buyer’s independent valuation can cause “frenzies” and “crashes.”} However, as discussed in the conclu-
sion, we believe that informational cascades are also important in settings in which individuals are trying to forecast others’ actions.\(^{34}\)

VI. Concluding Remarks

Conformity often appears spontaneously without any obvious punishment of the deviators. Informational cascades can explain how such social conventions and norms arise, are maintained, and change. We show that cascades can explain not only conformity but the rapid spread of new behaviors. We argue that conformist behaviors can be fragile and idiosyncratic because cascades start readily on the basis of even a small amount of information. There are many models that have unstable equilibria for some parameter values; this leads to fragility only when players happen to balance at a knife-edge. In our model, fragility arises systematically because cascades bring about precarious equilibria. It may be fruitful to perform experimental tests of how cascades form and shift in order to gain insight into the process of social change.

It should be noted that actual applications usually involve mixtures of informational effects, sanctions against deviants, payoff externalities, and conformity preference. All these are important for understanding social behavior. Nevertheless, even behavior that has been explained by sanctions, payoff externalities, or conformity preference may often be better understood with an analysis that combines these mechanisms with informational cascades.

Some of these alternative theories may permit the existence of multiple equilibria. If everyone expects others to switch to another equilibrium, then it pays people to conform to the change. Therefore, if people are primed for change, perhaps by a central authority, then the action can shift. For example, consumers may expect a new season to introduce new clothing fashions. Similarly, Kuran (1989, 1991) points out that once people believe that the government will fall, individuals become more willing to voice opposition publicly. Either of these scenarios can be combined with cascade effects. Sequential observation of decisions of previous individuals can lead to a cascade on a new fashion or a political revolution. Cascades can explain the process by which society switches from one equilibrium to another.

A generalization of our model is to allow individuals to invest in obtaining information. Similar results apply in this setting, with individuals obtaining information only until a cascade starts and later individuals imitating the early ones.

\(^{34}\) We conjecture that other types of noise or shocks, such as imperfect observation of actions or ignorance of preferences, can also shift cascades and cause fads.
An important extension of the model would allow individuals to have heterogeneous but correlated values of adoption. In such a setting, since others' decisions are less relevant, cascades may begin later and be less likely to occur. If the types of individuals are not common knowledge, an adoption cascade may occur, but the action may be undesirable for those who have a lower value of adoption. These individuals may still adopt because they do not know perfectly their predecessors' personal values of adoption.

Another possible extension of the analysis would examine liaison individuals, that is, individuals who link two or more cliques (see Rogers [1983] for a discussion). For example, a cascade in France may go in the opposite direction from a cascade in Great Britain. If one individual can observe both cascades and if only his decision can be observed in both countries, then he may break one of the two cascades. As the world becomes more of a global village, our analysis predicts that such linkage can reverse local cascades. U.S. "cultural imperialism" (in television, cinema, fast food, sneakers, and blue jeans) may be a case in point. Socially, it may be desirable to have separate groups that are only later combined, so that later individuals can aggregate the information of several cascades instead of just one.

Appendix

The proof of proposition 1 requires the following strong law of large numbers (see DeGroot 1970, p. 203).

**Strong Law of Large Numbers.** Let $Z_1, Z_2, \ldots$ be a sequence of independent and identically distributed random variables with mean $\lambda$. Then, with probability one,

$$
\lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} Z_j = \lambda.
$$

**Proof of Proposition 1**

We now prove that, for all $\epsilon > 0$, there exists an integer $N(\epsilon)$ such that with probability $1 - \epsilon$ a cascade starts at or before period $N(\epsilon)$.

The monotone likelihood ratio property implies that the conditional distributions of the signals are ordered by strict first-order stochastic dominance, that is,

$$
P_{q_1} > P_{q_2} > \ldots > P_{q_R} \quad \text{for all } q < R.
$$

Further, assumption 1 implies that, for all $A_n$, $V_{n+1}(x_q; A_n)$ is increasing in $q$.

To simplify the notation, throughout the rest of the proof we assume that $R = 3$. The proof for $R > 3$ is very similar. The proof for $R = 2$ is a special case of a result in DeGroot (1970, pp. 202–4).

Suppose that a cascade has not started till period $n + 1$. It is easily verified that each of the preceding individuals made use of their private signal, and
their actions indicate whether their private signal was greater than or less than a cutoff value. Let the history of actions be \( A_n \). Then for each \( i \), either \( j_i(A_{i-1}, \text{adopt}) \in \{x_2, x_3\} \) or \( j_i(A_{i-1}, \text{adopt}) \in \{x_3\} \). For each \( l = 1, 2, \ldots, S \), let \( Z_{q_l}, q = 1, 2 \), be binomial random variables with distribution \( \text{prob}\{Z_{q_l} = 0\} = P_{q_l} \) and \( \text{prob}\{Z_{q_l} = 1\} = 1 - P_{q_l} \). Thus if the true value of \( V \) is \( v_{l\star} \), then the information revealed to individual \( n + 1 \) by his predecessors’ actions consists of \( n_q \) \((q = 1, 2)\) realizations of \( Z_{q\star} \), where \( n_q \geq 0 \), \( n_1 + n_2 = n \). The realizations of \( Z_{q\star} \) are indexed \( Z_{q\star}(j) \), where \( j = 1, \ldots, n_q \). These \( n \) realizations are independent. Of course, individual \( n + 1 \) does not know the true value and therefore does not know whether he has observed \( n_q \) realizations of \( Z_{q_1} \), or \( Z_{q_2} \) or \( \ldots \) \( Z_{q\star} \) or \( \ldots \) or \( Z_{qS} \). Consequently, individual \( n + 1 \)’s posterior distribution conditional on the actions of his predecessors, \( A_n \), and on his signal realization \( X_{n+1} = x_q \) is

\[
\text{prob}\{V = v_l|A_n, X_{n+1} = x_q\} = \frac{\mu_l\prod_{j=1}^{n_1} \text{prob}\{Z_{1\star}(j)\} \prod_{j'=1}^{n_2} \text{prob}\{Z_{2\star}(j')\}}{\sum_{l'=1}^{S} \mu_{l'}\cdot P_{q_{l'}} \prod_{j=1}^{n_1} \text{prob}\{Z_{1\star}(j)\} \prod_{j'=1}^{n_2} \text{prob}\{Z_{2\star}(j')\}}.
\]

(A2)

where

\[
\text{prob}\{Z_{q\star}(j)\} = \begin{cases} P_{q_l} & \text{if } Z_{q_l}(j) = 0 \\ 1 - P_{q_l} & \text{if } Z_{q_l}(j) = 1. \end{cases}
\]

We show that if \( V = v_{l\star} \) then with probability one

\[
\lim_{n \to \infty} \text{prob}\{V = v_l|A_n, X_{n+1} = x_q\} = \begin{cases} 1 & \text{if } l = l\star \\ 0 & \text{if } l \neq l\star. \end{cases}
\]

(A3)

The proof of (A3) is a simple extension of an argument in DeGroot (1970, pp. 202–4). Let

\[
\lambda_{q_l} = E\left[ \log \left( \frac{\text{prob}\{Z_{q_l}\}}{\text{prob}\{Z_{q\star}\}} \right) | V = v_{l\star} \right].
\]

If \( l \neq l\star \), then (A1) and Jensen’s inequality imply that

\[
\lambda_{q_l} < \log \left\{ E\left[ \frac{\text{prob}\{Z_{q_l}\}}{\text{prob}\{Z_{q\star}\}} \right] | V = v_{l\star} \right\}
\]

\[
= \log \left[ \frac{P_{q_l}}{P_{q_l\star}} + \frac{1 - P_{q_l}}{1 - P_{q_l\star}} \right]
\]

\[
= \log 1
\]

\[
= 0.
\]

Thus the strong law of large numbers implies that with probability one

\[
\lim_{n_q \to \infty} \frac{1}{n_q} \sum_{j=1}^{n_q} \log \left( \frac{\text{prob}\{Z_{q\star}(j)\}}{\text{prob}\{Z_{q\star}(j)\}} \right) = \lambda_{q_l} < 0.
\]
Hence

$$\log \lim_{n_k \to \infty} \prod_{j=1}^{n_k} \frac{\text{prob}(Z_q(j))}{\text{prob}(Z_{q*}(j))} = \lim_{n_k \to \infty} \sum_{j=1}^{n_k} \log \left( \frac{\text{prob}(Z_q(j))}{\text{prob}(Z_{q*}(j))} \right) = -\infty.$$ 

Consequently, if \( l \neq l^* \), then, with probability one,

$$\lim_{n_k \to \infty} \prod_{j=1}^{n_k} \frac{\text{prob}(Z_q(j))}{\text{prob}(Z_{q*}(j))} = 0. \quad (A4)$$

If \( n \to \infty \) in (A3), then since \( n_1 + n_2 = n \), either \( n_1 \to \infty \) or \( n_2 \to \infty \) or both. Hence, (A4) and (A2) imply (A3). Therefore, if \( V = v_{l^*} \), then, for all \( \epsilon > 0 \),

$$\lim_{m \to \infty} \text{prob}\{ |V_{n+1}(X_{n+1}; A_n) - v_{l^*}| \leq \epsilon, \forall n \geq m \} = 1.$$

Thus assumption 2 implies that as \( m \to \infty \), with probability one either individual \( m \) and all subsequent individuals will adopt regardless of their private signal realization, or individual \( m \) and all subsequent individuals will reject regardless of their private signal realization. Q.E.D.

If assumption 2 is violated and \( v_l = C \) for some \( l \), then (a proof similar to that of proposition 1 establishes that) a cascade starts if \( V \neq v_l \). If \( V = v_l = C \), then the expectation of \( V \) conditional on the actions of individuals \( i < n \) approaches \( C \) as \( n \to \infty \). Thus the expectation of \( V \) conditional on the actions of individuals \( i < n \) and on \( X_n \) may be below \( C \) for lower realizations of \( X_n \) and above \( C \) for higher realizations of \( X_n \). Therefore, a cascade may not begin.

\textbf{Proof of Result 2}

The proof is an example that shows that if there is an information release, the second individual after the information is released is worse off. Let \( V \) be either zero or one with equal probability and the cost of adoption be .555. Let there be three signal values, \( X_i \in \{ x_1, x_2, x_3 \} \), with conditional distribution as listed in table A1. It can be readily verified that this example satisfies assumptions 1 and 2.

First consider the case in which no public information is released. Tables A2 and A3 list the probability that \( V = 1 \) given the first and second individuals’ information sets, the unconditional probability of these information sets (i.e., the ex ante probability that a particular signal is observed), and the individuals’ decisions. Column 1 of table A2 lists the unconditional probability that individual 1 observes the given signal; Columns 2 and 3 list the posterior expected value of adoption (the probability that \( V = 1 \)) and the resulting action of this individual.

Thus individual 1 adopts if \( X_1 \in \{ x_2, x_3 \} \) and rejects if \( X_1 = x_1 \). Individual 2 observes this decision, \( D_1 \in \{ A, R \} \) (adopt/reject) and his own signal, as described in table A3. Individual 2’s ex ante expected profits without public information can be computed from table A3 to be .445(.63 - .555) + .055(.73 - .555) = .0425.

Now consider the following release of public information. Both individuals observe a signal, \( S \), that is either high (\( H \)) or low (\( L \)) with probabilities \( \text{Pr}(S = H | V = 1) = .51 \) and \( \text{Pr}(S = H | V = 0) = .49 \).

Tables A4 and A5, analogous to tables A2 and A3, describe the decision problems of individual 1 and individual 2, respectively. As table A4 shows,
TABLE A1

**Conditional Signal Probability**

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Pr(X_i</td>
<td>V = 0)$</td>
<td>.4</td>
<td>.55</td>
</tr>
<tr>
<td>$\Pr(X_i</td>
<td>V = 1)$</td>
<td>.2</td>
<td>.7</td>
</tr>
</tbody>
</table>

TABLE A2

**The Decision of Individual 1 without Public Information Release**

<table>
<thead>
<tr>
<th></th>
<th>Unconditional Probability (1)</th>
<th>Expected Value (2)</th>
<th>Action (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>.3000</td>
<td>.3333</td>
<td>reject</td>
</tr>
<tr>
<td>$x_2$</td>
<td>.6250</td>
<td>.5600</td>
<td>adopt</td>
</tr>
<tr>
<td>$x_3$</td>
<td>.0750</td>
<td>.6667</td>
<td>adopt</td>
</tr>
</tbody>
</table>

the public information changes individual 1's decision to reject if $X_1 = x_2$ and $S = L$. Thus individual 2 can no longer infer that the signal of individual 1 was $x_1$ when he observes rejection. This changes the decision problem of individual 2. From table A5, the changed ex ante expected proceeds of the second individual with public information release can be computed to be $0.0242(0.71 - 0.555) + 0.0031(0.794 - 0.555) + 0.0339(0.6454 - 0.555) + 0.22365(0.638 - 0.555) + 0.02775(0.735 - 0.555) = 0.03114$, which is less than the ex ante expected proceeds without public information release ($0.0425$). Q.E.D.

**Proof of Result 4**

From equation (1), the probability that a cascade has not started by $n = 100$ is less than $1/2^{100} = 0$. Thus, by symmetry,

$$\Pr(\text{up cascade by } n = 100) = \Pr(\text{down cascade by } n = 100) \approx \frac{1}{2}. \quad (A5)$$

Also, equation (3) implies that

$$\Pr(V = 1|\text{up cascade started in period } 2n) = \Pr(V = 0|\text{down cascade started in period } 2n) = \frac{p(p + 1)}{2(p^2 - p + 1)} > \frac{1}{2} \quad (A6)$$

and

$$\Pr(V = 0|\text{up cascade started in period } 2n) = \Pr(V = 1|\text{down cascade started in period } 2n) = \frac{(2 - p)(1 - p)}{2(p^2 - p + 1)} < \frac{1}{2}. \quad (A7)$$

Recall that $W$ denotes the true value after $i = 100$. Let OD refer to the event that the new value $W$ is determined by the old drawing of $V$ (probability .9),
### TABLE A3
**The Decision of Individual 2 without Public Information Release**

<table>
<thead>
<tr>
<th>( D_1, X_2 )</th>
<th>Unconditional Probability (1)</th>
<th>Expected Value (2)</th>
<th>Action (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R, x_1 )</td>
<td>.1000</td>
<td>.2000</td>
<td>reject</td>
</tr>
<tr>
<td>( R, x_2 )</td>
<td>.1800</td>
<td>.3839</td>
<td>reject</td>
</tr>
<tr>
<td>( R, x_3 )</td>
<td>.0200</td>
<td>.5000</td>
<td>reject</td>
</tr>
<tr>
<td>( A, x_1 )</td>
<td>.2000</td>
<td>.4000</td>
<td>reject</td>
</tr>
<tr>
<td>( A, x_2 )</td>
<td>.4450</td>
<td>.6292</td>
<td>adopt</td>
</tr>
<tr>
<td>( A, x_3 )</td>
<td>.0550</td>
<td>.7273</td>
<td>adopt</td>
</tr>
</tbody>
</table>

### TABLE A4
**The Decision of Individual 1 with Public Information Release**

<table>
<thead>
<tr>
<th>( S, X_1 )</th>
<th>Unconditional Probability (1)</th>
<th>Expected Value (2)</th>
<th>Action (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L, x_1 )</td>
<td>.151</td>
<td>.3245</td>
<td>reject</td>
</tr>
<tr>
<td>( L, x_2 )</td>
<td>.31175</td>
<td>.55012</td>
<td>reject</td>
</tr>
<tr>
<td>( L, x_3 )</td>
<td>.03725</td>
<td>.657718</td>
<td>adopt</td>
</tr>
<tr>
<td>( H, x_1 )</td>
<td>.149</td>
<td>.34228</td>
<td>reject</td>
</tr>
<tr>
<td>( H, x_2 )</td>
<td>.31325</td>
<td>.5698</td>
<td>adopt</td>
</tr>
<tr>
<td>( H, x_3 )</td>
<td>.03775</td>
<td>.6755</td>
<td>adopt</td>
</tr>
</tbody>
</table>

### TABLE A5
**The Decision of Individual 2 with Public Information Release**

<table>
<thead>
<tr>
<th>( S, D_1, X_2 )</th>
<th>Unconditional Probability (1)</th>
<th>Expected Value (2)</th>
<th>Action (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L, A, x_1 )</td>
<td>.01</td>
<td>.49</td>
<td>reject</td>
</tr>
<tr>
<td>( L, A, x_2 )</td>
<td>.0242</td>
<td>.71</td>
<td>adopt</td>
</tr>
<tr>
<td>( L, A, x_3 )</td>
<td>.0031</td>
<td>.794</td>
<td>adopt</td>
</tr>
<tr>
<td>( L, R, x_1 )</td>
<td>.14176</td>
<td>.313</td>
<td>reject</td>
</tr>
<tr>
<td>( L, R, x_2 )</td>
<td>.2847</td>
<td>.537</td>
<td>reject</td>
</tr>
<tr>
<td>( L, R, x_3 )</td>
<td>.0339</td>
<td>.6454</td>
<td>adopt</td>
</tr>
<tr>
<td>( H, A, x_1 )</td>
<td>.0996</td>
<td>.4096</td>
<td>reject</td>
</tr>
<tr>
<td>( H, A, x_2 )</td>
<td>.22365</td>
<td>.638</td>
<td>adopt</td>
</tr>
<tr>
<td>( H, A, x_3 )</td>
<td>.02775</td>
<td>.735</td>
<td>adopt</td>
</tr>
<tr>
<td>( H, R, x_1 )</td>
<td>.0494</td>
<td>.2065</td>
<td>reject</td>
</tr>
<tr>
<td>( H, R, x_2 )</td>
<td>.0896</td>
<td>.3984</td>
<td>reject</td>
</tr>
<tr>
<td>( H, R, x_3 )</td>
<td>.01</td>
<td>.5100</td>
<td>reject</td>
</tr>
</tbody>
</table>
and let ND refer to the event of a new drawing (probability .1). By (A6),

\[
\Pr(W = 1 \mid \text{up cascade by } n = 100) = \Pr(W = 0 \mid \text{down cascade by } n = 100)
\]

\[
= \Pr(\text{OD}) \times \frac{\rho (\rho + 1)}{2(\rho^2 - \rho + 1)} + \Pr(\text{ND}) \times \frac{1}{2}
\]

\[
= .9 \frac{\rho (\rho + 1)}{2(\rho^2 - \rho + 1)} + .05
\]

\[> \frac{1}{2}, \quad (A8)\]

for all \( \rho > \frac{1}{2} \).

In order to examine the decisions of the 101st and later individuals, we condition on new information \((X_{101}, X_{102})\). First, we calculate the probability that an up cascade started by period 100 is reversed. Let \(E[W \mid \text{up}, H]\) denote \(E[W \mid \text{up cascade by } n = 100, X_{101} = H]\). Clearly,

\[
E[W \mid \text{up}, H] > E[W \mid \text{up}] = \Pr(W = 1 \mid \text{up}) > \frac{1}{2}, \quad (A9)
\]

where the last inequality follows from (A8). Therefore, after an up cascade, if \(X_{101} = H\), then individual 101 adopts. Next, let \(\Pr(v, w, L \mid \text{up})\) denote the conditional probability that \(V = v, W = w, \) and \(X_{101} = L\) given that an up cascade started before \(n = 101\):

\[
E[W \mid \text{up}, L]
\]

\[
= \Pr(W = 1 \mid \text{up}, L)
\]

\[
= \frac{\Pr(W = 1, L \mid \text{up})}{\Pr(L \mid \text{up})}
\]

\[
= \frac{\Pr(1, 1, L \mid \text{up}) + \Pr(0, 1, L \mid \text{up})}{\Pr(1, 1, L \mid \text{up}) + \Pr(0, 1, L \mid \text{up}) + \Pr(1, 0, L \mid \text{up}) + \Pr(0, 0, L \mid \text{up})},
\]

To illustrate how these terms are calculated, we use (A6) to express \(\Pr(1, 1, L \mid \text{up})\) as

\[
\Pr(1, 1, L \mid \text{up}) = \Pr(V = 1 \mid \text{up}) \Pr(W = 1 \mid V = 1, \text{up}) \Pr(L \mid V = 1, W = 1, \text{up})
\]

\[
= \Pr(V = 1 \mid \text{up}) \Pr(W = 1 \mid V = 1) \Pr(L \mid W = 1)
\]

\[
= \frac{\rho (\rho + 1)}{2(\rho^2 - \rho + 1)} \cdot .95(1 - \rho).
\]

Thus, using (A6) and (A7), we have

\[
E[W \mid \text{up}, L]
\]

\[
= \left[ \frac{\rho (\rho + 1)}{2(\rho^2 - \rho + 1)} \cdot .95(1 - \rho) + \frac{(2 - \rho)(1 - \rho)}{2(\rho^2 - \rho + 1)} \cdot .05(1 - \rho) \right]
\]

\[
+ \left[ \frac{\rho (\rho + 1)}{2(\rho^2 - \rho + 1)} \cdot .95(1 - \rho) + \frac{(2 - \rho)(1 - \rho)}{2(\rho^2 - \rho + 1)} \cdot .05(1 - \rho) \right] + \frac{\rho (\rho + 1)}{2(\rho^2 - \rho + 1)} \cdot .05p + \frac{(2 - \rho)(1 - \rho)}{2(\rho^2 - \rho + 1)} \cdot .95p
\]

\[
= \frac{(1 - \rho)(2 + 16\rho + 20\rho^2)}{2 + 52\rho - 52\rho^2}
\]

\[= .488\]
for $p = .9$. Hence, after an up cascade, if individual 101 observes $L$, he rejects. Subsequent individuals can infer $X_{101}$ from individual 101’s action.

It is easily established that $E[W|\text{up}, L, H] = E[W|\text{up}]$, and thus (A9) implies that $E[W|\text{up}, L, H] > \frac{1}{2}$. Hence, if individual 101 rejects, that is, $X_{101} = L$, then 102 adopts if $X_{102} = H$. On the other hand, if individual 102 observes $L$ after 101 rejects, then 102 rejects because

$$E[W|\text{up}, L, L] < E[W|\text{up}, L] < \frac{1}{2},$$

where the last inequality follows from (A10). Thus if 101 rejects, then $X_{102}$ is inferred from 102’s action ($H$ if adopt, $L$ if reject). Moreover, if 101 and 102 reject, a down cascade starts since

$$E[W|\text{up}, L, L, H] = E[W|\text{up}, L] < \frac{1}{2}.$$ 

The intuition for these results is that a cascade contains little information to start with, so new information can easily reverse the information reflected in the old cascade. Thus the conditional probability of a change in convention to down after $n = 100$ given an up cascade initially is at least

$$Pr(\text{down after } n = 100|\text{up before } n = 101) \geq Pr(X_{101} = L, X_{102} = L|\text{up})$$
$$= .9[Pr(V = 1|\text{up})Pr(L, L|V = 1, \text{up}) + Pr(V = 0|\text{up})Pr(L, L|V = 0, \text{up})]$$
$$+ .1 Pr(L, L|\text{ND})$$
$$= .9\left[\frac{p(p + 1)(1-p)^2 + (2-p)(1-p)p^2}{2(p^2 - p + 1)}\right] (A11)$$
$$+ .1[\frac{1}{2} Pr(L, L|W = 1) + \frac{1}{2} Pr(L, L|W = 0)]$$
$$= .9\left[\frac{p(1-p)(1 + 2p - 2p^2)}{2(p^2 - p + 1)}\right] + .05[(1-p)^2 + p^2]$$
$$= .0935$$

for $p = .9$.

A symmetric argument establishes that a down cascade before $n = 100$ is reversed if $X_{101} = X_{102} = H$ and that

$$Pr(\text{up after } n = 101|\text{down before } n = 101) \geq Pr(X_{101} = H, X_{102} = H|\text{up}) = .0935. (A12)$$

Hence, for $p = .9$, a lower bound on the probability of a cascade reversal after period 101 is

$$Pr(\text{cascade reversal after 100})$$
$$\geq Pr(X_{101} = L, X_{102} = L|\text{up before 101})Pr(\text{up before 101})$$
$$+ Pr(X_{101} = H, X_{102} = H|\text{down before 101})Pr(\text{up before 101})$$
$$= .0935 \times \frac{1}{2} + .0935 \times \frac{1}{2}$$
$$= .0935,$$

where the first equality follows from (A5), (A11), and (A12). Q.E.D.
References


